

Scale Invariance as a Solution to the Cosmological Constant Problem

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Abstract: We show that scale invariance provides a solution to the fine tuning problem of the cosmological constant. We construct a generalization of the standard model of particle physics which displays exact quantum scale invariance. The matter action is invariant under global scale transformations in arbitrary dimensions. However the gravitational action breaks scale invariance explicitly. The scale symmetry is broken spontaneously in the matter sector of the theory. We show that the contribution to the vacuum energy and hence the cosmological constant is identically zero from the matter sector within the full quantum theory. However the gravitational sector may give non-zero contributions to the cosmological constant at loop orders. No fine tuning may be required at loop orders since the matter sector gives zero contribution to the cosmological constant. We also show that we do not require full scale invariance in order to constrain the vacuum energy from the matter sector. We only require invariance under pseudoscale transformations. Using this idea and motivated by the concept of unimodular gravity we propose an alternative model. In this case also we show that matter sector gives exactly zero contribution to the vacuum energy.

1 Introduction

The standard model of particle physics generically gives rise to a very large value of vacuum energy and hence suggests the presence of a cosmological constant many orders of magnitude larger than what is allowed by the observations. This has to be canceled by explicitly introducing a large cosmological constant term of opposite sign, leading to a fine tuning problem [1–7]. Different approaches for solving the cosmological constant problem have been considered by several authors [1, 8–17]. We consider the idea of scale invariance. It has the potential to

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provide an answer to the fine tuning problem associated with the cosmological constant. Scale invariance naturally rules out a cosmological constant [18, 19].

In recent papers [19–23] we have studied a scale invariant generalization of the Standard Model of particle physics, including gravity [24]. The model contains no dimensionful parameter and displays local scale invariance – an idea first proposed by Weyl back in the twenties [25] and later explored by many others [26–46]. In the model considered in Refs. [19–23], scale invariance is broken by the phenomenon which we call as cosmological symmetry breaking [19, 47]. Here, motivated by the Big Bang cosmology, we assume a spatially flat background Friedmann-Robertson-Walker (FRW) metric. The model admits a classical time dependent solution which breaks scale invariance. This generates all the dimensionful parameters in the theory such as the Hubble parameter, the Planck mass, the vacuum or dark energy, Weyl meson mass as well as the electro-weak particle masses [19, 21–23].

In this paper we consider a model where the matter sector displays global scale invariance. The gravitational term in the action, however, breaks scale invariance. Before electro-weak symmetry breaking, the matter sector is invariant under global scale transformation as it contains no dimensionful parameter. This requires the introduction of an additional real scalar field apart from the standard Higgs field. The scale invariance is ultimately broken spontaneously along with the electro-weak symmetry. The gravitational action is the standard Hilbert-Einstein action with an explicit cosmological constant and hence violates scale invariance. For the model considered in Ref. [23], it has been shown that pure gauge fields lead to vanishing cosmological constant at all orders in the gauge field coupling. This follows for all the gauge fields which do not have any coupling to the Higgs particle. Similar result applies to fermion fields, as long as we ignore their coupling to Higgs. All of these results are applicable for the model considered in the present paper. However the problem of fine tuning of the cosmological constant was addressed only partially in Ref. [23] as we were not able to explicitly rule out the possibility of large contributions from cosmological constant from the matter sector. The basic problem arose due to the fact that Higgs field was coupled non-minimally to gravity. However, no such problem exists in the present model and we demonstrate that due to exact scaling symmetry in the matter action cosmological constant receives no contribution from the matter sector.

Scale invariance is generally believed to be anomalous. However it has been shown that scale transformations can be extended to arbitrary dimensions such that the action is invariant under this generalized transformation [19, 21–23, 48–50]. Within dimensional regularization, the scale anomaly arises primarily since the action is not invariant under scale transformations in dimensions other than four [51]. In the present case the matter action is made to be scale invariant in arbitrary dimensions by introducing terms with fractional powers of the additional scalar field. Hence, as long as we neglect gravitational corrections, once the action is specified in d dimensions, we can directly extract dimensionally regularized Feynman amplitudes for the matter sector [52]. However, we note that this theory makes sense only if the scalar field, which is raised to a fractional power, is non-zero classically. In this case one may make an expansion around this classical value and the theory is well defined. However the theory is ill defined if the minimum of the potential arises at zero values of the classical scalar fields. If, for some reason, one demands that only terms with integral powers have to appear in the action, then

the theory necessarily has scale anomaly.

As an alternative, we also consider a model based on the idea of unimodular gravity [1, 9, 10, 12–15]. Here we suggest a model which does not respect the full general coordinate transformations but only their unimodular subset. The matter sector of this theory is invariant under pseudoscale transformations [22, 24] but not under the standard scale transformations. We show that this is sufficient to constrain the contribution to the cosmological constant from the matter sector.

The present paper is organized as follows. In section 2 we describe the model and discuss the allowed ranges of various parameters of the model. In section 3 we compute the contribution of the matter sector to the cosmological constant. In section 4 we expand around the classical solution to display the particle spectrum of this theory. In section 5 we discuss the alternative model based on the idea of unimodular gravity and finally in section 6 we conclude.

2 A Model with Scale Invariant Matter Action

Consider the following action in $d = 4 - \epsilon$ dimensions containing gravity, a scalar field Φ and the standard model fields,

$$\mathcal{S} = \int d^d x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{16\pi} R - \Lambda + \mathcal{L}_{\text{Matter}} \right], \quad (1)$$

where Λ is the cosmological constant, M_{Pl} is the Planck's constant and $\mathcal{L}_{\text{Matter}}$ refers to the matter Lagrangian which includes the standard model fields and the additional real scalar field Φ . We choose the conventions followed in Refs. [53, 54] where the flat space-time metric takes the form $(1, -1, -1, -1)$. The curvature tensor and its contractions are defined as,

$$\begin{aligned} R_{\nu\alpha\beta}^{\mu} &= -\partial_{\beta}\Gamma_{\nu\alpha}^{\mu} + \partial_{\alpha}\Gamma_{\nu\beta}^{\mu} + \Gamma_{\gamma\alpha}^{\mu}\Gamma_{\nu\beta}^{\gamma} - \Gamma_{\gamma\beta}^{\mu}\Gamma_{\nu\alpha}^{\gamma}, \\ R_{\nu\beta} &= R_{\nu\beta\mu}^{\mu}, \quad R = R_{\nu\beta}g^{\nu\beta}. \end{aligned} \quad (2)$$

The matter Lagrangian is given by,

$$\begin{aligned} \mathcal{L}_{\text{Matter}} &= g^{\mu\nu}(D_{\mu}\mathcal{H})^{\dagger}(D_{\nu}\mathcal{H}) + \frac{1}{2}g^{\mu\nu}\partial_{\mu}\Phi\partial_{\nu}\Phi - \frac{\lambda_1}{4}(\mathcal{H}^{\dagger}\mathcal{H} - \lambda_2\Phi^2)^2(\Phi^2)^{-\delta} \\ &\quad - \frac{1}{4}g^{\mu\nu}g^{\alpha\beta}(\mathcal{A}_{\mu\alpha}^i\mathcal{A}_{\nu\beta}^i + \mathcal{B}_{\mu\alpha}\mathcal{B}_{\nu\beta} + \mathcal{G}_{\mu\alpha}^j\mathcal{G}_{\nu\beta}^j)(\Phi^2)^{\delta} + \mathcal{L}_{\text{fermions}}, \end{aligned} \quad (3)$$

where $\delta = (d-4)/(d-2)$, \mathcal{H} is the Higgs doublet, $\mathcal{G}_{\mu\nu}^j$, $\mathcal{A}_{\mu\nu}^i$ and $\mathcal{B}_{\mu\nu}$ represent the field strength tensors for the $SU(3)$, $SU(2)$ and $U(1)$ gauge fields of the standard model respectively. The superscripts i and j on $\mathcal{A}_{\mu\nu}^i$ and $\mathcal{G}_{\mu\nu}^j$ represent the $SU(2)$ and $SU(3)$ indices respectively. All the repeated indices are implicitly summed over. The scalar potential of this model is identical to what is proposed in Ref. [49]. The fermionic Lagrangian in d -dimensions is given by,

$$\mathcal{L}_{\text{fermions}} = \left(\bar{\Psi}_{\text{L}} i\gamma^{\mu} \mathcal{D}_{\mu} \Psi_{\text{L}} + \bar{\Psi}_{\text{R}} i\gamma^{\mu} \mathcal{D}_{\mu} \Psi_{\text{R}} \right) - \left[g_Y \bar{\Psi}_{\text{L}} \mathcal{H} \Psi_{\text{R}} (\Phi^2)^{-\delta/2} + h.c. \right], \quad (4)$$

where $\gamma^\mu = e^\mu_a \gamma^a$, e^μ_a is the vierbein field and a, b are Lorentz indices. Here Ψ_L is an $SU(2)$ doublet, Ψ_R a singlet and g_Y represents the Yukawa coupling for the field Ψ . For simplicity we have displayed only one Yukawa coupling term and included only one family. In Eq. 4, D_μ represents the covariant derivative of the fermion field,

$$\mathcal{D}_\mu \Psi_{L/R} = \left(\tilde{D}_\mu + \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} \right) \Psi_{L/R} , \quad (5)$$

where $\tilde{D}_\mu \Psi_L = \partial_\mu - ig \mathbf{T} \cdot \mathbf{A}_\mu - ig'(Y_f^L/2)B_\mu$, $\tilde{D}_\mu \Psi_R = \partial_\mu - ig'(Y_f^R/2)B_\mu$, $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ and ω_μ^{ab} represents the spin connection. Here A_μ is the $SU(2)$ field, B_μ the $U(1)$ field, \mathbf{T} represents the $SU(2)$ generators and Y_f the $U(1)$ hypercharges. The color interaction of quarks has been suppressed for simplicity.

The terms such as $(\Phi^2)^\delta$ which involve the scalar field raised to a fractional power are handled by making a Taylor expansion around the classical value of the field [19, 21–23, 48–50]. Hence these make sense only if the classical value is non-zero. If the minimum of the potential arises at zero value of the classical field then the model is ill defined. If the classical value of the scalar field Φ is non-zero then these terms provide an infinite series of terms involving higher and higher powers of the fluctuating fields. These higher order terms might cause some complications at loop orders, as discussed below and in Refs. [49, 55].

The scaling symmetry of the action in Eq. 1 is spoiled by the presence of the dimensionful parameters, M_{PL} and Λ , in the gravitational sector of the model. The cosmological constant may, in principle, also acquire contributions at loop orders from the matter sector. However, as we shall show, the matter sector gives identically zero contribution to the vacuum energy and hence to the cosmological constant since the corresponding action respects scale invariance. We point out that due to the presence of the terms with fractional powers of the scalar field Φ it displays global scale invariance in arbitrary dimensions. Hence within the framework of dimensional regularization, all the regularized Feynman amplitudes respect scale invariance.

In our model both the scalar fields acquire non zero classical solutions and the scaling symmetry in the matter action is broken spontaneously. The potential is minimized if [49],

$$\mathcal{H}^\dagger \mathcal{H} = \lambda_2 \Phi^2. \quad (6)$$

We assume that classically, [49],

$$\Phi = \Phi_0, \quad (7)$$

where Φ_0 is a dimensionful parameter. We point out that Eqs. 6 and 7 solve the classical equations of motion. We denote the classical Higgs field by \mathcal{H}_0 , *i.e.*,

$$\mathcal{H}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (8)$$

where v is the electro-weak symmetry breaking scale.

In $d = 4$, the extra scalar field Φ couples directly only to the Higgs field but in any other

dimension it couples to other matter fields via the fractional powers of Φ in different terms of the matter Lagrangian. Hence it may contribute at loop orders. As in earlier papers [19, 21–23, 48–50], the scalar field terms raised to fractional powers, such as $(\Phi^2)^{-\delta}$, are handled by making an expansion around their classical values,

$$\Phi = \Phi_0 + \hat{\Phi}. \quad (9)$$

For any process the higher order terms in this expansion give contributions at least of order p^2/Φ_0^2 where p is the momentum scale of the process under consideration. Hence the theory is perturbatively reliable only up to the scale Φ_0 [55]. The parameter Φ_0 has to be chosen suitably so that perturbation theory is valid for sufficiently large mass scale compared to the electro-weak scale. As the two parameters, λ_2 and Φ_0 , are related to the electro-weak scale as,

$$v^2 = 2\lambda_2\Phi_0^2, \quad (10)$$

for large Φ_0 , the parameter, λ_2 has to be very small to generate the correct electro-weak scale. It has been shown in Ref. [49] that a small value of λ_2 does not lead to any fine tuning problem. Hence for Φ_0 sufficiently larger than the electro-weak scale, the model is free of fine tuning and is consistent with present observations. One may, of course, consider a supersymmetric generalization of this model which may provide an alternate mechanism to stabilize the scalar potential.

Although in general scale invariance would allow terms with three independent parameters in the potential term of the scalar fields, we have included only two. The reason for this particular choice of potential term is discussed in detail in Ref. [49], where the authors obtain the one loop effective action corresponding to this model. If we include all the possible terms then the potential does not have a minimum at nonzero values of the scalar fields. In this case the theory is not phenomenologically acceptable and is also theoretically ill defined due to the fractional powers of the field Φ . However, the fact that one of the allowed terms is not present in the action implies that we may not be able to absorb all the divergences into renormalization of the parameters of the model. This term will be generated at loop orders. In order to cancel this we may require introduction of additional terms involving the scalar fields, spoiling the renormalization of the model. The renormalization of the model may also be spoiled by higher order terms in the expansion of terms such as $(\Phi^2)^{-\delta}$. However even if the theory is not renormalizable, phenomenologically it is not a serious problem [49, 55], since the theory is perturbatively reliable only at low energies, $E \ll \Phi_0$, where the additional contributions are very small. Furthermore we point out that it is possible to introduce additional terms in the action which vanish in the limit $d = 4$. For example, in full generality we may replace,

$$(\Phi^2)^{-\delta} \rightarrow [\Phi^2 + \lambda_3 \mathcal{H}^\dagger \mathcal{H}]^{-\delta} \quad (11)$$

in Eq. 3. This introduces an additional parameter λ_3 which might be useful in canceling some of the divergences in the scalar field sector of this theory. The entire issue of renormalizability of these models is more complicated and we postpone it to further research.

Before we proceed to compute the contribution of the matter sector to the cosmological constant a comment on the gravitational action is in order. Although the gravitational action breaks scale invariance in this model, it is possible to generalize this action to make it scale invariant by introducing an additional scalar field and setting $\Lambda = 0$. In this case scale invariance in the gravitational sector could be broken by a mechanism we call cosmological symmetry breaking [19–23] where a classical time dependent solution breaks the scale invariance. Assuming an FRW metric, we find an exact solution with the scale parameter expanding exponentially with time [19]. In this case the Planck constant as well as the Hubble constant are generated dynamically [19]. The theory also generates an effective cosmological constant. This mechanism has some similarity to the phenomenon of phase transitions induced by background curvature [56–68] but is not identical. A theory where the entire action is scale invariant is aesthetically appealing. However the action displayed in Eq. 1 is simpler to handle and closer to the Einstein's gravity. We postpone a detailed analysis of the scale invariant gravitational action to a future paper.

3 Contribution to the Cosmological Constant

The important point about this model is that the matter Lagrangian displays exact scale invariance in arbitrary dimensions. The Lagrangian of Eq. 3 will directly lead to dimensionally regulated amplitudes. As the scale current is conserved in arbitrary dimensions, we do not expect any scale anomaly [19, 21–23, 48–50]. This implies that the trace of the energy momentum tensor must vanish and there would be no contribution to the cosmological constant from the matter part of the action. We next verify this explicitly.

The energy momentum tensor following from the d dimensional action may be expressed as,

$$\begin{aligned} T_{\alpha\beta} = & -g_{\alpha\beta}\mathcal{L} + (D_\alpha\mathcal{H})^\dagger(D_\beta\mathcal{H}) + (D_\beta\mathcal{H})^\dagger(D_\alpha\mathcal{H}) \\ & + \frac{1}{2}\partial_\alpha\Phi\partial_\beta\Phi + \frac{1}{2}\partial_\beta\Phi\partial_\alpha\Phi - g^{\mu\nu}B_{\alpha\mu}B_{\beta\nu}(\Phi^2)^\delta \\ & + \frac{1}{2}\bar{\Psi}_L i\gamma_\alpha D_\beta \Psi_L + \frac{1}{2}\bar{\Psi}_R i\gamma_\alpha D_\beta \Psi_R + \frac{1}{2}\bar{\Psi}_L i\gamma_\beta D_\alpha \Psi_L + \frac{1}{2}\bar{\Psi}_R i\gamma_\beta D_\alpha \Psi_R, \end{aligned} \quad (12)$$

where we have explicitly displayed the contribution coming from only one vector field. The other vector fields give similar contributions. The equations of motion for the scalar fields are given by,

$$D^\mu D_\mu \mathcal{H} = -\frac{\lambda_1}{2}(\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)\mathcal{H}(\Phi^2)^{-\delta} - g_Y \bar{\Psi}_R \Psi_L (\Phi^2)^{-\delta/2}, \quad (13)$$

$$\begin{aligned} \Phi \partial^\mu \partial_\mu \Phi = & \lambda_1 \lambda_2 (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2) \Phi^2 (\Phi^2)^{-\delta} + \frac{\lambda_1 \delta}{2} (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)^2 (\Phi^2)^{-\delta} \\ & - \frac{\delta}{2} B^{\alpha\mu} B_{\alpha\mu} (\Phi^2)^\delta + \delta g_Y [\bar{\Psi}_L \mathcal{H} \Psi_R (\Phi^2)^{-\delta/2} + h.c.] \end{aligned} \quad (14)$$

and the equations of motion for the fermion fields may be written as,

$$i\gamma^\mu D_\mu \Psi_R - g_Y \mathcal{H}^\dagger \Psi_L (\Phi^2)^{-\delta/2} = 0, \quad (15)$$

$$i\gamma^\mu D_\mu \Psi_L - g_Y \mathcal{H} \Psi_R (\Phi^2)^{-\delta/2} = 0. \quad (16)$$

Using the fermion equations of motion, the trace of energy momentum tensor may be expressed as,

$$\begin{aligned} T_\alpha^\alpha &= \frac{1}{2}(d-2) \left[\mathcal{H}^\dagger D^\alpha D_\alpha \mathcal{H} + h.c. \right] + \frac{1}{2}(d-2) \Phi \partial^\alpha \partial_\alpha \Phi - B^{\alpha\mu} B_{\alpha\mu} (\Phi^2)^\delta \\ &+ d \left[\frac{\lambda_1}{4} (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)^2 (\Phi^2)^{-\delta} + \frac{1}{4} B^{\alpha\mu} B_{\alpha\mu} (\Phi^2)^\delta \right] \\ &+ g_Y \left[\bar{\Psi}_L \mathcal{H} \Psi_R (\Phi^2)^{-\delta/2} + h.c. \right], \end{aligned} \quad (17)$$

up to terms which are total derivatives. These will drop out after integration by parts and hence won't contribute to the cosmological constant. Now using the equations of motion for the scalar fields one finds that $T_\alpha^\alpha = 0$. Hence the matter sector of the theory gives zero contribution to the cosmological constant. The entire contribution to the cosmological constant comes from the gravitational sector of theory and hence no fine tuning may be required in fitting its value to observations.

We emphasize that Eq. 17 lists all the operators that can contribute to the trace of the energy momentum tensor in this theory after dimensional regularization. Hence the cosmological constant in the full quantum theory must vanish if the right hand side of Eq. 17 vanishes. In our demonstration that $T_\alpha^\alpha = 0$, we have used the classical equations of motion. However this is perfectly justified since these are valid as Heisenberg operator equations [69].

Apart from the terms in the Lagrangian in Eqs. 1, 3 and 4 we are allowed to introduce additional terms in the action such as, $\Phi^2 R$, $\mathcal{H}^\dagger \mathcal{H} R$ without violating the scaling symmetry. We assume that the couplings corresponding to these terms are negligible. Even at loop orders such couplings will be generated only by quantum gravity effects and hence will be suppressed.

The gravitational part of the action explicitly violates scale invariance. It may contribute to the cosmological constant at loop orders. The one loop and two loop contributions due to the gravitation have been considered earlier [70, 71]. However the precise result for the gravitational contribution to cosmological constant is unknown due to the usual problems in handling quantum gravity. Here we assume that these contributions are small and give only small corrections to the cosmological constant parameter introduced in our model.

4 Expansion Around the Classical Solution

The one loop effective action for this model has already been computed in Ref. [49]. Here we briefly consider the quadratic terms in the expansion around the classical solution, shown in Eqs. 7 and 8, in order to display the particle spectrum of this theory. We have,

$$\Phi = \Phi_0 + \hat{\Phi}, \quad \mathcal{H} = \mathcal{H}_0 + \hat{\mathcal{H}}. \quad (18)$$

We parametrize $\hat{\mathcal{H}}$ as,

$$\hat{\mathcal{H}} = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_1 + i\xi_2 \\ \xi_3 + i\xi_4 \end{pmatrix} \quad (19)$$

and denote the real Higgs field as $\hat{\mathcal{H}}_3 = v + \xi_3$. The electro-weak scale $v^2 = 2\lambda_2\Phi_0^2$. We next expand the potential in d dimensions around this classical solution. We have,

$$\begin{aligned} V &= \frac{\lambda_1}{4} (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)^2 (\Phi^2)^{-\delta} \\ &= \frac{\lambda_1}{4} (\Phi_0^2)^{-\delta} \left[(v\xi_3 - 2\lambda_2\Phi_0\hat{\Phi})^2 + \dots \right], \end{aligned} \quad (20)$$

where we display only the quadratic terms. The fields ξ_3 and $\hat{\Phi}$ mix with each other. We make an orthogonal transformation to diagonalize the mass matrix. The transformed fields are given by,

$$\begin{aligned} \phi_1 &= \cos \theta \xi_3 - \sin \theta \hat{\Phi}, \\ \phi_2 &= \sin \theta \xi_3 + \cos \theta \hat{\Phi}, \end{aligned} \quad (21)$$

where

$$\sin \theta = \sqrt{\frac{2\lambda_2}{1 + 2\lambda_2}} \approx 0. \quad (22)$$

From Eq. 20 we find that the field ϕ_1 has mass $m_1^2 = \lambda_1 v^2 (1 + 2\lambda_2)/2$. The field ϕ_2 acts as the Goldstone boson corresponding to spontaneous breakdown of scale symmetry with mass $m_2 = 0$.

The field ϕ_2 is essentially the field $\hat{\Phi}$. It couples indirectly to the matter fields through the Yukawa interaction terms. The coupling at tree level is zero. However we get a coupling proportional to the mass of matter particles at loop orders. Since this particle has zero mass it will have long range interactions and hence can lead to a modification in Newtonian gravity. The precise value of the correction depends on Φ_0 . If Φ_0 is comparable or much larger than the Planck mass then the correction may be small and negligible. In the present paper we do not consider the phenomenological implications of this particle. However it is clear that for sufficiently large value of Φ_0 its contribution will not violate the observational bounds.

5 Unimodular Gravity

In this section we shall slightly deviate from the main development of this paper and suggest an alternative model based on the idea of unimodular gravity [1, 9, 10, 12–15]. Motivated by the work of Refs. [12, 13, 72] and the discussion in Ref. [1] we consider the possibility that the quantum theory may not respect the full general coordinate invariance, $x^\mu \rightarrow x'^\mu$. It may respect only the restricted invariance where $\det(\partial x'^\mu / \partial x^\nu) = 1$. However the classical theory respects the full general coordinate transformations. In d dimensions the matter action in this case may be obtained by replacing $(\Phi^2)^\delta$ by $(\sqrt{-g})^{(4-d)/d}$ in Eq. 3 as,

$$\begin{aligned} \mathcal{L}_{\text{Matter}} &= g^{\mu\nu} (D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) + \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda_1}{4} (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)^2 (\sqrt{-g})^{(d-4)/d} \\ &- \frac{1}{4} g^{\mu\nu} g^{\alpha\beta} (\mathcal{A}_{\mu\alpha}^i \mathcal{A}_{\nu\beta}^i + \mathcal{B}_{\mu\alpha} \mathcal{B}_{\nu\beta} + \mathcal{G}_{\mu\alpha}^j \mathcal{G}_{\nu\beta}^j) (\sqrt{-g})^{(4-d)/d} + \mathcal{L}_{\text{fermions}}, \end{aligned} \quad (23)$$

with the fermion Lagrangian similarly modified. The dimensionally regulated Feynman amplitudes obtained from this Lagrangian will not obey the full general coordinate invariance. They will only obey the limited transformation rule under which $\det(\partial x'^\mu/\partial x^\nu) = 1$. They will also not obey scale invariance. Hence scale transformations are anomalous in the present model. However the model is invariant under pseudoscale transformations in d dimensions [22, 24]. Hence the dimensionally regulated Feynman amplitudes would obey this symmetry as long as we consider only the matter sector of the theory and neglect the gravitational corrections to the matter sector.

For completeness, here we review the pseudoscale transformations [22, 24]. In d dimensions the transformation law is as follows,

$$\begin{aligned} x &\rightarrow x, \\ \Phi &\rightarrow \Phi/\Omega, \\ g^{\mu\nu} &\rightarrow g^{\mu\nu}/\Omega^{b(d)}, \\ g_{\mu\nu} &\rightarrow g_{\mu\nu}\Omega^{b(d)}, \\ A_\mu &\rightarrow A_\mu, \\ \Psi &\rightarrow \Psi/\Omega^{c(d)}, \end{aligned} \tag{24}$$

where $b(d) = 4/(d-2)$ and $c(d) = (d-1)/(d-2)$. In Eq. 24 Φ represents any scalar field, A_μ a vector field and Ψ a spinor field. The action corresponding to the Lagrangian given in Eq. 23 is invariant under this transformation in d dimensions. Hence the dimensionally regulated Feynman amplitudes would preserve this symmetry. However the usual scale transformations, under which $g_{\mu\nu}$ does not change but $x \rightarrow \Omega x$ are anomalous. Hence the coupling λ_1 would be dimensionful in dimensions d different from 4, indicating breakdown of scale invariance. Similarly the gauge coupling would be dimensionful in dimensions d different from 4.

The cosmological constant or vacuum energy term, however, is not invariant under pseudoscale transformations. Hence we do not require the full scale transformations to constrain the vacuum energy contributions from matter sector. Pseudoscale transformations are sufficient for this purpose.

It is clear that the modified theory is consistent with all observations. Classically, in four dimensions, it produces same results as Einstein's gravity. It's predictions differ only when we consider quantum gravity effects which are very tiny at the energy scales probed by the current experiments. We point out that in contrast to the model described in Eq. 3, the present model is valid perturbatively when we expand around the trivial classical solution where the background classical fields are zero.

The matter sector of the model shows exact pseudoscale invariance [19, 22–24] in the full quantum theory. As discussed above the cosmological constant term violates pseudo-scale invariance. Hence, in analogy with the calculation in Section 3, we again expect to find zero contribution to the cosmological constant from the matter fields. This may also be verified explicitly. We have checked that the matter action produces vanishing T_μ^μ in d dimensions, exactly as was found in Section 3 for the model discussed in Section 2. Hence the matter fields produce zero vacuum energy at all orders in perturbation theory, as long as we ignore corrections due

to gravitational coupling.

The main phenomenological advantage of the action, Eq. 23, is that the couplings of Φ to the Standard Model fields are now extremely tiny. They will arise only due to its mixing with the Higgs boson which is proportional to λ_2 . Hence in this case we may allow a much wider range of values for the scale Φ_0 .

6 Conclusions

In this paper we have presented a generalization of the standard model of particle physics which displays exact quantum scale invariance except in the gravitational sector. We explicitly show that within the framework of dimensional regularization, the matter sector of the theory gives identically zero contribution to the cosmological constant. This contribution vanishes due to scale or pseudoscale invariance and applies to the full quantum theory. The entire contribution to the cosmological constant arises from the gravitational sector of theory. Hence the loop corrections are naturally suppressed and no fine tuning may be required to fit the observations. This shows that scale or pseudoscale invariance may provide a natural solution to the problem of cosmological constant.

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